Canada's national laboratory
for particle and nuclear physics
and accelerator-based science

SRG Evolution of $0\nu\beta\beta$ Operators in Light Nuclei

Peter Gysbers[1,2]
P. Navrátil[2] & S. Quaglioni[3]
[1]University of British Columbia, [2]TRIUMF, [3]LLNL

Feb 15, 2019

Outline

- Background
- Two-body operators
  - recap: β-decay, $0\nu\beta\beta$
  - progress: NCSM vs CC benchmarks in light nuclei
- Three-body operators
  - progress: implementing transition densities

- SRG evolution is a unitary transformation which improves convergence

$$U_\alpha \hat{O} U_\alpha^\dagger = O_\alpha^{(1)} + O_\alpha^{(2)} + O_\alpha^{(3)} + \dots$$

- Introduces higher-body terms, $O_\alpha^{(a)}$, determined in the appropriate a-body system ($a \leq A$)
- E.g. if $O = O^{(2)}$:

$$O_\alpha^{(2)} = U_\alpha^{(2)} O^{(2)} U_\alpha^{\dagger(2)}$$

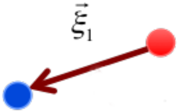
$$O_\alpha^{(3)} = U_\alpha^{(3)} \left\langle O^{(2)} \right\rangle^{(3)} U_\alpha^{\dagger(3)} - \left\langle O_\alpha^{(2)} \right\rangle^{(3)}$$

$$U_\alpha^{(2)} = \sum \left|\psi_{\alpha,a=2}\right\rangle \left\langle \psi_{a=2}\right|$$

$$U_\alpha^{(3)} = \sum \left|\psi_{\alpha,a=3}\right\rangle \left\langle \psi_{a=3}\right|$$

For $|\psi_k\rangle = |kj^\pi tt_z\rangle = \sum c^k_{n\ell s} |n\ell s j^\pi tt_z\rangle$, U_α is constructed in blocks:

$$U_\alpha^{j^\pi tt_z} = \sum_k |kj^\pi t, \alpha\rangle \langle kj^\pi t|$$



Non-scalar operators may connect states with $j^\pi tt_z$, e.g.

$$\langle f, j_f| O_\alpha |i, j_i\rangle = \langle f, j_f| U_\alpha^{j_f} O U_\alpha^{j_i \dagger} |j_i\rangle$$

Converting to single-particle basis:

$$\langle a'b'J_{a'b'}| O_\alpha |abJ_{ab}\rangle \qquad a = n_a, \ell_a, j_a$$

$$= \sum_{if} c^{if}_{a'b'ab} \langle f, j_f| O_\alpha |i, j_i\rangle$$

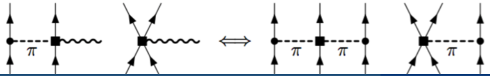
$$\hat{O} = GT^{(1)} + MEC^{(2)} \;\to\; \hat{O}_\lambda = GT^{(1)} + GT^{(2)}_\lambda + MEC^{(2)}_\lambda + \ldots$$

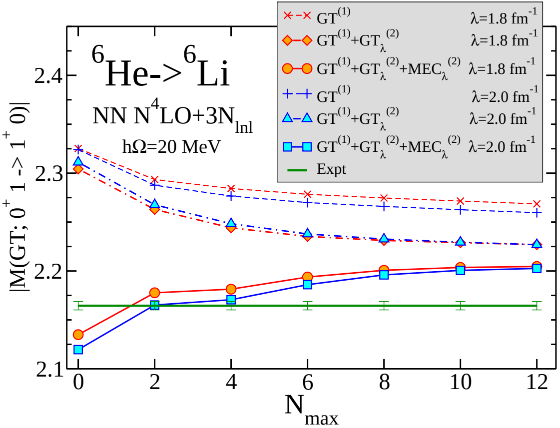
Operator:

Gamow-Teller (1-body) + chiral meson exchange current (2-body) Park (2003)

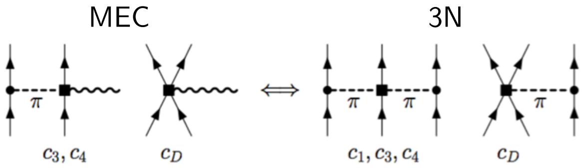
Potential: "N^{4}LO NN + $3N_{lnl}$"

- chiral NN @ N^{4}LO, Machleidt PRC96 (2017), 500MeV cutoff
- 3N local/non-local, Navrátil
- $c_D = -1.8$



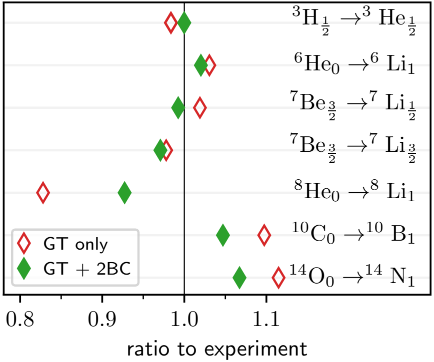


- SRG evolved matrix elements used in coupled-cluster and IM-SRG methods (up to Sn^{100})
- Does inclusion of the MEC explain g_A quenching?



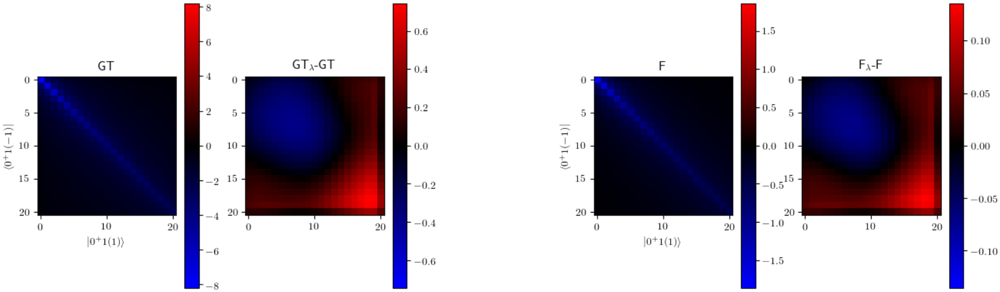
Gysbers et al, *Nature Physics* (accepted)

NCSM: NN-N^4LO+3N_{lnl}



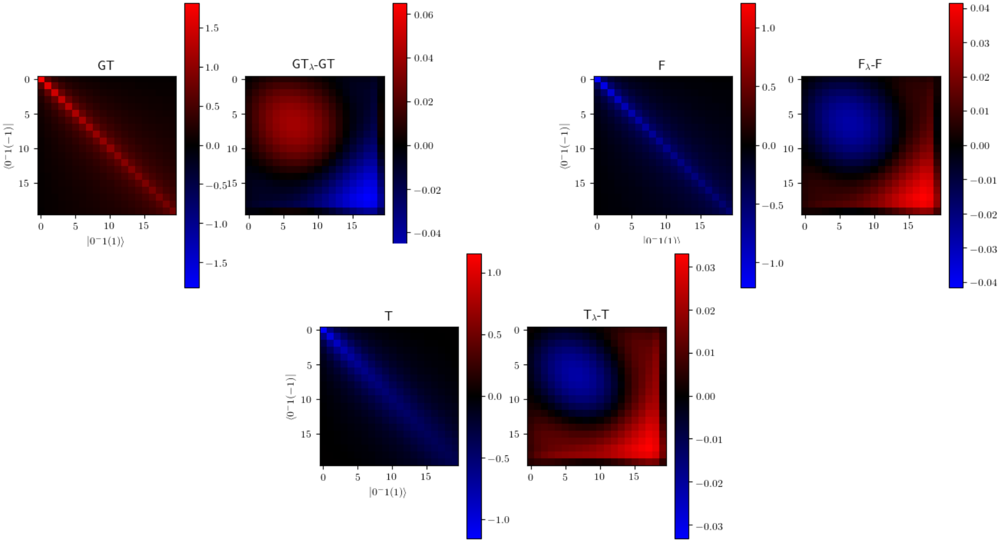
$$\hat{O}_{0\nu\beta\beta} = \hat{O}_{GT} + \hat{O}_F + \hat{O}_T \quad \text{from J. Engel}$$

$^1S_0$:



Two-body SRG: $\lambda = 2$ fm^{-1}, chiral NN @ N^{3}LO

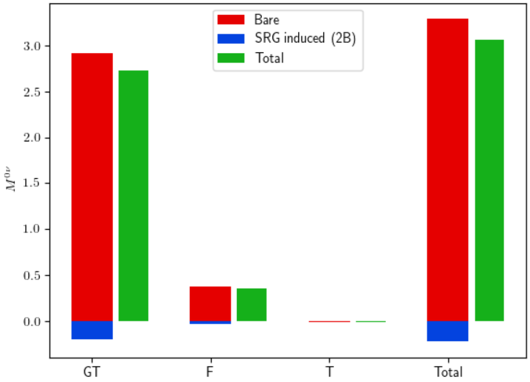
$^3P_0$:



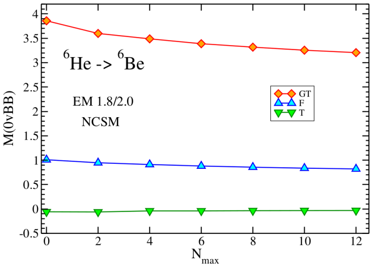
Two-body SRG: $\lambda = 2\ \mathrm{fm}^{-1}$, chiral NN @ N^3LO

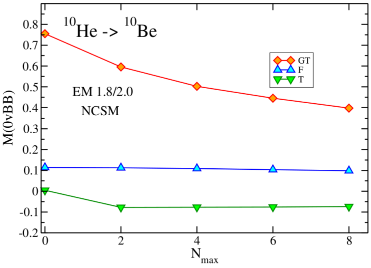
Preliminary Results: $0\nu\beta\beta_{\lambda,2b}$

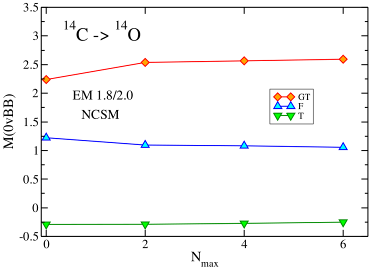
- Shell-model (J. Engel, M. Horoi)
- ^{76}Ge
- JUN45 interaction, $\hbar\omega = 9.23$ MeV
- $\sim 7\%$ effect, $\lambda = 2\ \mathrm{fm}^{-1}$

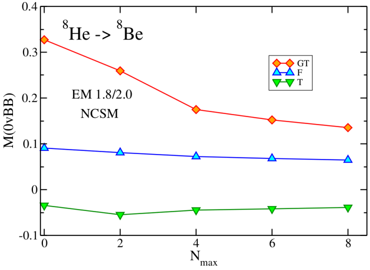


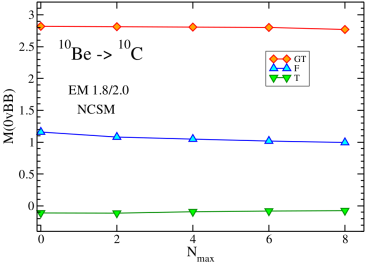
Results in Light Nuclei



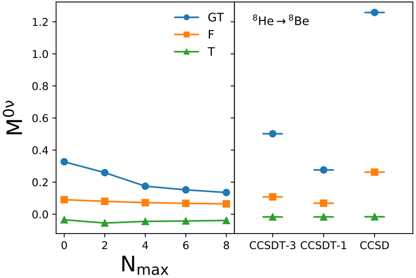




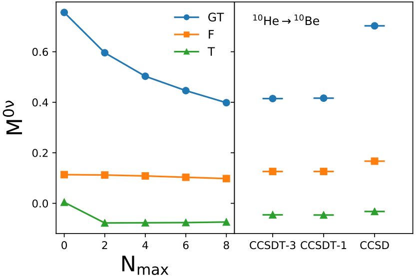


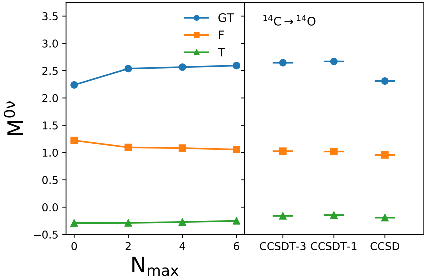


Seems consistent with GFMC (Pastore **PRC97** 014606 2018)

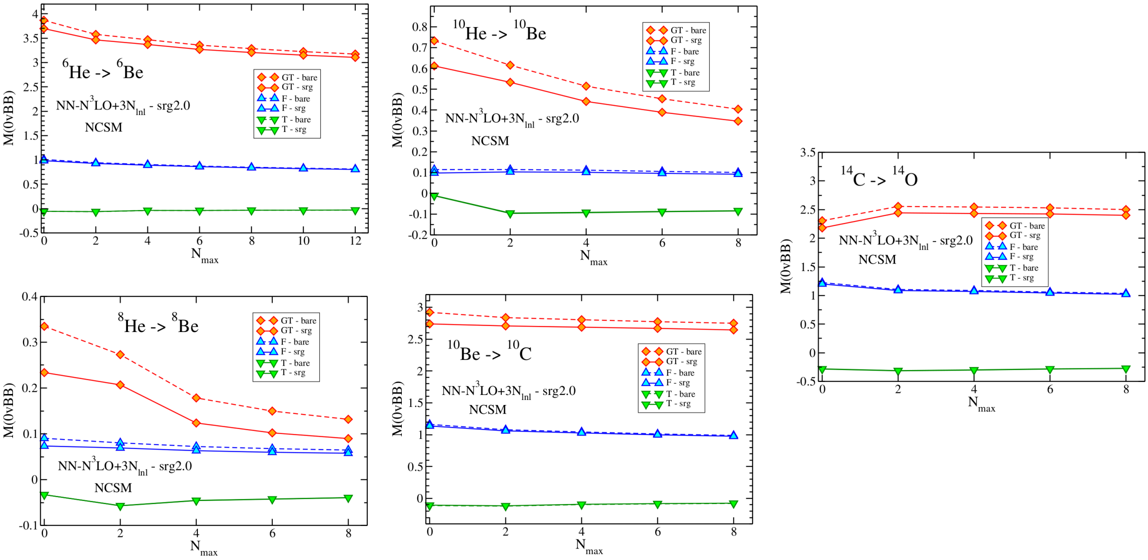


(Figures from Sam Novario)



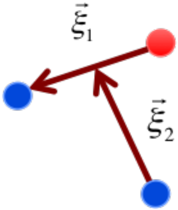


Effects of SRG (two-body)



For $|\psi_k\rangle = |kJ^\rho T\rangle = \sum c^k_{Ni} |NiJ^\rho T\rangle$
$|NiJ^\rho T\rangle = \sum \mathcal{C}^{NiJT}_{n\ell sjt;\mathcal{NLJ}} |(n\ell sjt;\mathcal{NLJ})JT\rangle \quad (N = 2n + \ell + 2\mathcal{N} + \mathcal{L})$

$$U^{J^\rho T}_\alpha = \sum_k |kJ^\rho T, \alpha\rangle \langle kJ^\rho T|$$



Non-scalar operators may connect states with $J^\rho T(T_z)$, e.g.

$$\langle f, J_f| O_\alpha |i, J_i\rangle = \langle J_f| U^{J_f}_\alpha O U^{J_i \dagger}_\alpha |i, J_i\rangle$$

Converting to single-particle basis:

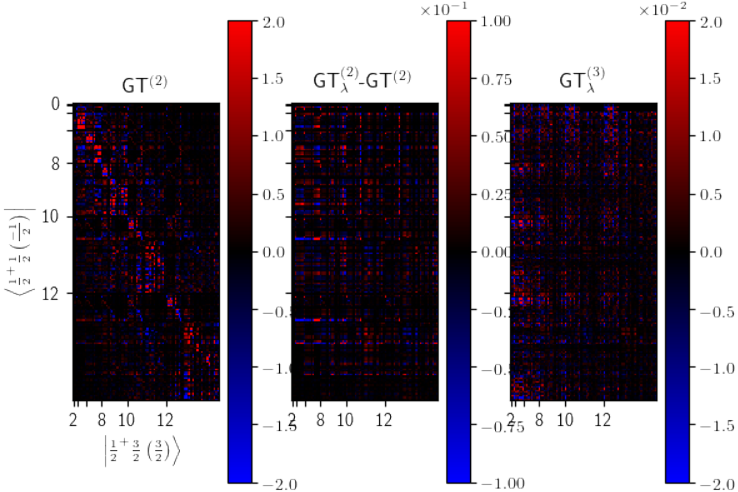
$$\langle a'b'J_{a'b'}c'J_{a'b'c'}| O_\alpha |abJ_{ab}cJ_{abc}\rangle \qquad a = n_a, \ell_a, j_a$$
$$= \sum_{if} c^{if}_{a'b'c'abc} \langle f, J_f| O_\alpha |i, J_i\rangle$$

- Generalized code to calculate:

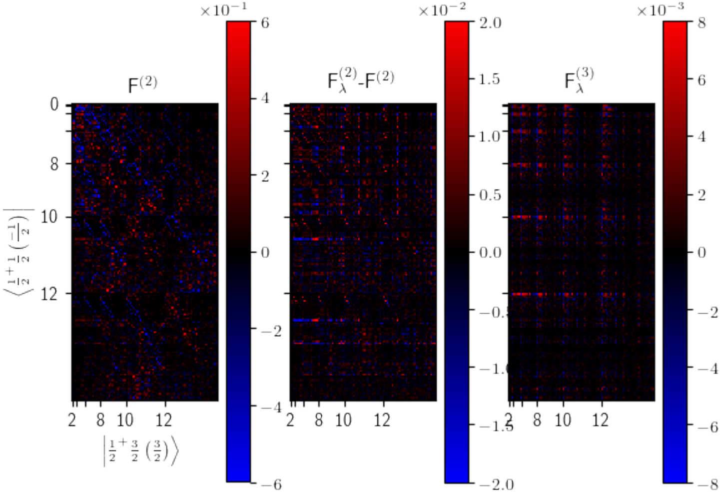
$$\langle f, J_f || \hat{O}^{(3)} || i, J_i \rangle = \frac{1}{36} \sum \langle \alpha\beta\gamma | \hat{O} | \delta\epsilon\omega \rangle \langle f, J_f || a^\dagger_\alpha a^\dagger_\beta a^\dagger_\gamma a_\omega a_\epsilon a_\delta || i, J_i \rangle$$

- Decouple $\langle abJ_{ab}cJ_{abc} | \hat{O} | deJ_{de}fJ_{def} \rangle \rightarrow \langle abc | \hat{O} | def \rangle$ on the fly
- Benchmarked general operator method with three-body interaction

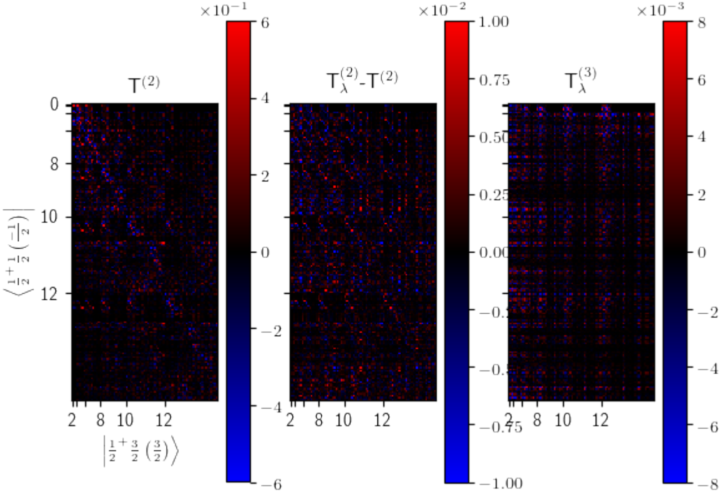
Three-body SRG:
$\lambda = 2\ \text{fm}^{-1}$
NN@N^{3}LO



Three-body SRG:
$\lambda = 2\ \mathrm{fm}^{-1}$
NN@N^{3}LO



Three-body SRG:
$\lambda = 2\ \mathrm{fm}^{-1}$
NN@N^{3}LO



Summary and Outlook

- Operators must be SRG evolved to converge to the correct result
- Method implemented in 2B and 3B for arbitrary operators
- So far: $\sigma\tau$, axial MEC, $0\nu\beta\beta$, radius, E2
- Available in single-particle coordinates
- Results for β-decay strengths: ^{3}H$\rightarrow$^{3}He, ^{6}He$\rightarrow$^{6}Li and other nuclei
- Results for $0\nu\beta\beta_{\lambda,2b}$: ^{8}H$\rightarrow$^{8}Be, ^{14}C$\rightarrow$^{14}O, etc
- In progress:
  - Application of $0\nu\beta\beta_{\lambda,3b}$ matrix elements in many-body methods
  - Quantification of 2- and 3-body evolution effects

$$\hat{O}_{0\nu\beta\beta} = \hat{O}_{GT} + \hat{O}_F + \hat{O}_T$$

$$O_\gamma = H_\gamma y_\gamma \tau_1^+ \tau_2^+$$

$$H_\gamma(r_{12}) = \frac{2R}{\pi} \int_0^\infty \mathrm{d}q \frac{q \cdot f_\gamma(q \cdot r_{12}) h_\gamma(q^2)}{q + E_0^{\mathrm{cl}}}$$

$$y_\gamma = \begin{cases} 1 & \gamma = F \\ \sigma_1 \cdot \sigma_2 & \gamma = GT \\ \sqrt{\frac{24\pi}{5}} Y_2(\hat{r_{12}}) \left(3(\sigma_1 \cdot r_{12})(\sigma_2 \cdot r_{12}) - \sigma_1 \cdot \sigma_2\right) & \gamma = T \end{cases}$$

Goal: solve the nuclear eigenvalue problem

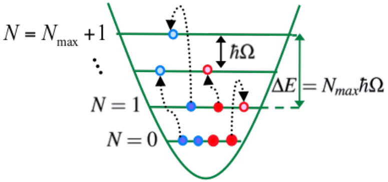
$$H|\Psi_k\rangle = E_k|\Psi_k\rangle, \text{ where } H = \sum_i^A T_i + \sum_{i<j} V_{ij} + \sum_{i<j<f} V_{ijf} + \cdots$$

with nucleons as the degrees of freedom

The No-core Shell Model

Expand in anti-symmetrized products of harmonic oscillator single-particle states

$$|\Psi_k\rangle = \sum_{N=0}^{N_{max}} \sum_j c^k_{Nj} |\Phi_{Nj}\rangle$$



Calculations should converge to the exact value as $N_{max} \to \infty$

- Problem: Huge model-space size required to accommodate short-range physics
- Solution: use renormalized potentials in smaller model-space
- Caveat: need renormalized operators

Unitary transformation that decouples high and low momentum physics

$$H_\alpha = U_\alpha H U_\alpha^\dagger \text{ where } U_\alpha U_\alpha^\dagger = 1$$

$$\frac{\mathrm{d}H_\alpha}{\mathrm{d}\alpha} = [\eta_\alpha, H_\alpha]$$

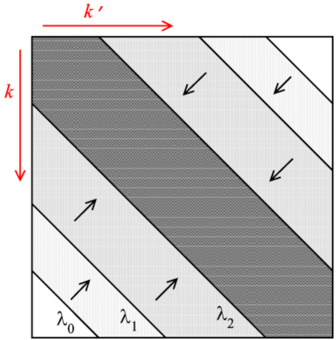
$$\eta_\alpha = \frac{\mathrm{d}U_\alpha}{\mathrm{d}\alpha} U_\alpha^\dagger = -\eta_\alpha^\dagger$$

Choose a generator, e.g. $\eta_\alpha = [T, H_\alpha]$

$$\lambda = \alpha^{-1/4}$$

$$H_{\lambda=\infty} = H, \quad U_{\lambda=\infty} = 1$$

$$H_{\alpha=0} = H, \quad U_{\alpha=0} = 1$$



Rep.Prog.Phys. **76** 126301 (2013)

General Operators

$$H |\Psi_k\rangle = E_k |\Psi_k\rangle \rightarrow H_\alpha |\Psi_{k,\alpha}\rangle = E_k |\Psi_{k,\alpha}\rangle$$

General operators must also be transformed:

$$\langle\Psi_f| \hat{O} |\Psi_i\rangle = \langle\Psi_{f,\alpha}| \hat{O}_\alpha |\Psi_{i,\alpha}\rangle \text{ where } \hat{O}_\alpha = U_\alpha \hat{O} U_\alpha^\dagger$$

$$U_\alpha = \sum_k |\Psi_{k,\alpha}\rangle \langle\Psi_k|$$

SRG transformations introduce higher-body terms in operators:

$$U_\alpha \hat{O} U_\alpha^\dagger = \hat{O}_\alpha^{(1)} + \hat{O}_\alpha^{(2)} + \hat{O}_\alpha^{(3)} + \dots$$

Each term, $\hat{O}_\alpha^{(a)}$, must be determined in the appropriate a-body system ($a \leq A$).
E.g. if $O = O^{(2)}$:

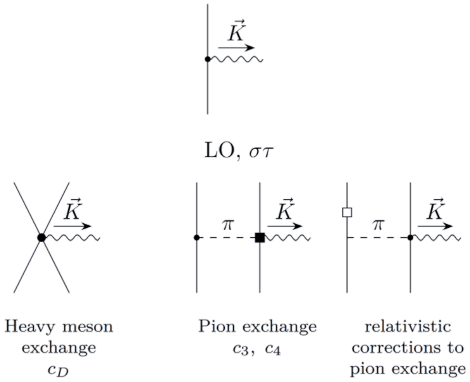
$$O_\alpha^{(2)} = U_\alpha^{(2)} O^{(2)} U_\alpha^{\dagger(2)}$$

$$O_\alpha^{(3)} = U_\alpha^{(3)} \left\langle O^{(2)} \right\rangle^{(3)} U_\alpha^{\dagger(3)} - \left\langle O_\alpha^{(2)} \right\rangle^{(3)}$$

$$U_\alpha^{(2)} = \sum |\psi_{\alpha,a=2}\rangle \langle \psi_{a=2}|$$

$$U_\alpha^{(3)} = \sum |\psi_{\alpha,a=3}\rangle \langle \psi_{a=3}|$$

- leading order $\sigma\tau$ (1-body): "Gamow-Teller" (GT)
- higher order (2-body): "Axial Meson Exchange Current" (MEC)
- shared parameters with chiral potentials



Results: β-decay $^3H \rightarrow ^3He$

$$\hat{O} = GT^{(1)} \rightarrow \hat{O}_\lambda = GT^{(1)} + GT_\lambda^{(2)} + \ldots$$

$$\lambda = \alpha^{-1/4} \quad [\text{fm}^{-1}]$$

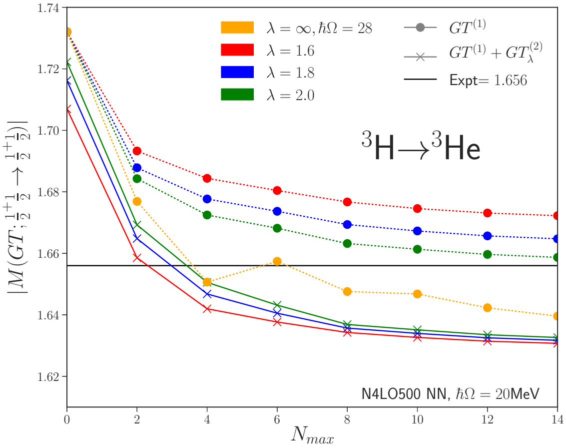
Operator:

Gamow-Teller (1-body)

$$\langle GT_\lambda^{(2)} \rangle_{A=2} = \langle (GT^{(1)})_\lambda \rangle_{A=2} - \langle GT^{(1)} \rangle_{A=2}$$

Potential: "N^{4}LO NN"

- chiral NN @ N^{4}LO, Machleidt PRC96 (2017), 500MeV cutoff



Results: β-decay $^3H \to {}^3He$

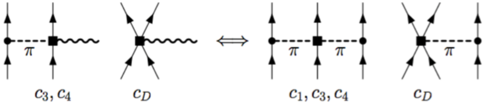
$$\hat{O} = GT^{(1)} + MEC^{(2)} \rightarrow \hat{O}_\lambda = GT^{(1)} + GT^{(2)}_\lambda + MEC^{(2)}_\lambda + \dots$$

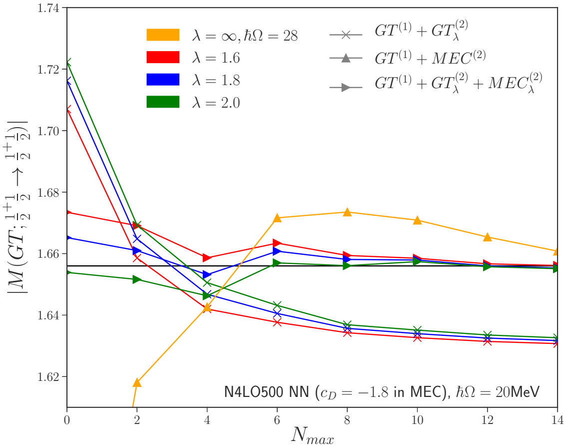
Operator:

Gamow-Teller (1-body) + chiral meson exchange current (2-body) Park (2003)

Potential: "N^{4}LO NN"

- chiral NN @ N^{4}LO, Machleidt PRC96 (2017), 500MeV cutoff
- LEC $c_D = -1.8$ determined





χEFT potentials and MECs share parameters

- c_D: one-pion exchange + 2N contact 3N force
- c_3, c_4: two-pion exchange 2N and 3N forces
- ^{3}He β-decay constrains c_D, insensitive to 3N force
- PRL **103** 102502 (2009) D. Gazit, S. Quaglioni, P. Navratil
- Errata: missing factor of $-\frac{1}{4}$ in MEC c_D term